Modelling of Price Dynamic and Depreciation[1]

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Abstract

This paper includes a model of price movements (market values) based on the general premise that price is equivalent to market value, as well as other premises. A concrete case involving the application of the dynamic theory of models to a description of the behaviour of price and use in a perfect competitive market is analyzed. The model is constructed from a first-order linear ordinary differential equation and shows, given the market assumptions referred to above for certain types of commodity (such as furs or textiles), that the theory conforms closely to practice, which can be used to construct an estimation theory (for details see Drozen, 2003).

Keywords: *model, price, value, market value, value in exchange, equation, differential equation*

JEL Classification: C00, D40

The economist is a man who knows the price of everything and the value of nothing.

Oscar Wilde

Introduction

The theory of prices and the attempts to predict their development have always belonged among the most difficult tasks of economists. Whether or not we like it, policy has always been the strongest link between the economist's cabinet and the real world. Economy is in this sense an examination of choice rather than

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a clear guide what should be chosen. That is also the case of the basic phenomena, such as the price. The price is a complex phenomenon, which is constituted by the supply and demand in all aspects, starting with the historical background of a community and ending with the result of the most recent football match. The price, which is determined incorrectly or wrongly, no matter why, is the basis of potential and useless collisions; that is why we should pay more attention to the basic terms of economy, such as the value and the price, also from the theoretical point of view. If we admit that the theoretical aspect is useful also for practice, there is a short step only to create model situations describing the given phenomenon.

Objective: To model deterministically the development of price in time for the commodities having a negative first derivation of price by time. That means, to find a deterministic equation describing the falling own value of commodity in time. It is known that papers dealing with the development of value as a unit in time use the stochastic differential equation of the development of price in time. By abstracting the probability part from the value of commodities in the general concept, the deterministic equation can be used to describe the development of the own value of a general commodity.

Initial assumption: Price = market value. The following definition can be used for practical purposes – "Market Value is the estimated amount for which a property should exchange on the date of valuation between a willing buyer and a willing seller (it represents the value in exchange). The parties are expected to act prudently and without compulsion."[2] (According to the International Valuation Standards – IVC).

The author is aware of the potential debate that might arise over the relationship between values[3] and prices, however, within the framework of a model differential equation describing price falls, this assumption, i.e. making prices equivalent to market values, is a fundamental premise. In other words we abstract values as such from theory (The concept of values is the foundation on which the science of Economics is built and is the key to solving economic problems (the well-known point of view of Eugene von Böhm-Bawerk, representative of the Austrian school of economics, who lived from 1851 to 1914). The Böhm-Bawerk's theory of the market price constitution indicates that price equals value (see Böhm-Bawerk, 1991, p. 109) (or rather that price equals the objective exchange value). In a more detailed analysis Böhm-Bawerk concludes

[2] International Valuation Standards 2001, Europen Valuation Standards 2001.

[3] The Aristotelian-scholastic tradition understands values (good, beauty, truth) as what it calls transcendentalia, i.e. as a designation whose level of universality is higher than the level of universality of the categories of substances, quality, quantity and relations. This means that values are what allow us to understand and explain, not what may and must be explained.

that “The objective exchange value is actually – as we can ascertain later – the result of subjective assessments of many individuals. The concept of exchange value is in a close relation to price; however, it does not equal price. Exchange value is the ability to get a quantum of other assets; price is that quantum of assets” (Böhm-Bawerk, 1991, p. 110). Actually, the author is in a tight corner there, trying to support the initial assumption (price = market value) of the traditional economic theories. It is an assumption; when quantifying or using the mathematic tools, we cannot do without assumptions.

Böhm-Bawerk differs (in the first plan) between the subjective value and the objective value: “Value in its subjective sense is the significance the asset or set of assets has for the subject's wealth (Wohlfahrtszwecke).”[4] Böhm-Bawerk also claims that “the value of an asset is determined by the size of its limit utility”. Whereas value in its objective sense means “the asset's strength or ability to cause some objective success or result (Erfolg)”.

According to Böhm-Bawerk, the exchange value is an objective value. (Böhm--Bawerk speaks directly about “the objective exchange value of assets”.) “This should be understood as objective validity of assets during exchange, or in other words, the possibility to get certain amount of other economic assets in exchange for them; the possibility means the strength or ability of the former assets.”

Within the framework of his subjectivism (which is typical for the Austrian school), Böhm-Bawerk stresses that both values – including the exchange value – are kinds of subjective value.

The objective exchange value is the *result of subjective assessments of many individuals.* [5]

As already mentioned, Böhm-Bawerk claims that the concept of exchange value “is in a close relation to price”; however, it does not equal price because “exchange value is the ability to get a quantum of other assets; price is that quantum of assets”.

In spite of that fine categorical differentiation, Böhm-Bawerk believes that if “the law of the prices of assets explains that and why the asset actually achieves a certain price, it also explains unintentionally that and why it is able to achieve a certain price”. That means the price law contains also the law of exchange value.

[4] In this sense I say that an asset is of value for me if I know that my wealth is associated with it in such a way that holding of the asset satisfies my needs, gives me pleasure I would miss, or spares me suffering I would have to undergo if I had not held that asset. In this case the existence of the asset means higher wealth for me, the loss of it means lower wealth (Lebenswohlfahrt); the asset is important for me, it is of value for me.

[5] Use value is the significance an asset has for the certain person's wealth providing that the person uses the asset directly for his/her needs; and analogically, the exchange value is the significance an asset achieves for the certain person's wealth due to its ability to get other assets in exchange.

As regards the amount of the market price (as determined by that "price law"), Böhm-Bawerk claims that "with bilateral competition the market price shall stabilize within the space, which is limited at the top by the assessments of the last buyer who still managed to exchange and the exchange of the aptest excluded willing seller; at the bottom the space is limited by the assessments of exchange of the least apt seller who managed to exchange and the exchange of the aptest willing buyer who is excluded from the exchange".

That can be understood as follows; the exchange value (or the possibility/ability to get a quantum of other assets in exchange for certain goods) in the form of price (or a certain quantum of other assets) realizes *adequately* only if the market price is constituted under the condition of pure competition, i.e. in line with the Böhm-Bawerk's model of horse market with bilateral competition. (Intervention by a central planner – non-market intervention – would cause a difference between the exchange value and the price.)

We can accept the simplified concept that the exchange (or objective) value of a commodity equals its price under the condition of perfect competition.[6]

Mathematic modelling below deals with potential falls of price (which equals the objective exchange value, i.e. the market value, in our simplified concept) in time.

Basic idea: Let us assume following generalization of the equation (2): "The proportionality constant" is a slowly-changing function of time, which is expressed as $k*f(t)$ (k is a constant number). If function f is developed as a function of one real variable, which is time (t) in this case, in the Taylor's line round time $t = 0$, we get the power line with the terms t^n $(n = 0, 1, 2\ldots)$. If this infinite series is limited to a finite number of addends, we get certain approximation of the real status of the development of price, and adding gradually further terms of the development of function f, we approximate the actual status. That is why the paper deals with the model approaches depending on the zero, first, square and third power of time.

The objective of this paper is to *model* deterministically the development of price in time for the commodities having a negative first derivation by time. Much was written about modelling. Let us mention only.

A special group of models are represented by what are called *mathematical models* (sometimes also less accurately referred to as logical models). *Representing reality by mathematical means* (i.e. using equations, inequalities, various types of equation and inequality systems) undoubtedly brings many advantages. The representation of elements such as dependencies, connections, causality or

[6] For other aspects of Böhm-Bawerk's theory of objective value see Pavlík (2004, pp. 673 – 676.)

mutual relations between two or more quantities using a mathematical model gives users an invaluable practical tool, which can be used not only for theoretical abstractions, but also to support concrete decisions or deductions. A mathematical model, in the form of an equation for example, also enables us to "substitute" variable values, which appear in the model as "causes" of the variable values or variables standing for the "effect". These independent or explanatory variables as they are known remain under our control in any given problem, and their values are therefore known or easy to extrapolate. The basic idea is that we can easily define the values of certain variables, or that we can define them more easily than the values of other variables that we may currently be interested in. Only by using the relationships and interdependencies of these two groups of variables are we able to convert the philosophical and theoretical problem of assessing these functionalities into the statistical problem of projecting the probability parameters of the model. Mathematical models may also help learning because they provide, or at least indicate, "patterns" that are generally not measurable, but still continue to be expressed through concrete sets of observations. In this way they can inspire us and provide incentives for further investigation. Thus, if the model is a *simplified abstraction of reality,* this means the mathematical model is also simplified. The specificity of mathematical models only resides in the means they employ, and occasionally in the aims they serve.

The saying that "all models are bad, but some are useful" is also fully applicable to mathematical models.

The text below is devoted to modelling the fall in the market value of goods on a general level. The market value of goods is here equal to the price of the goods.

1. Differential Equation of Price and Depreciation

In mathematical modelling of the market value of goods over time, expressed as a price, our first approximation proceeds from the assumption that the speed of reduction of the market value is directly proportionate to the price of the goods. The price of goods *n* over time *t*, i.e. *n(t)* falls by value Δn over the time elapsed Δt. The average speed of change $\overline{v}$ (change per unit of time) is derived from the fraction $\overline{v} = \frac{\Delta n}{\Delta t}$. To arrive at the immediate speed of change *v* we have to select the smallest possible interval of time and a limit conversion gives us the relationship:

$$v = \frac{dn(t)}{dt} = n'(t) \tag{1}$$

As a first approximation, this means that we will consider the speed of price changes to be directly proportionate to the amount of the price of the goods *n(t)* over time *t*.

In mathematical terms, this is expressed as:

$$\frac{dn(t)}{dt} = -kn(t) \tag{2}$$

where $k(k \rangle 0)$ is a constant of proportionality and the minus sign expresses the assumption that, with an increase in the time factor, *n(t)* diminishes.

The relationship (2) shown above is a first order linear differential equation and because of this the solution can be obtained by separating the variables. The initial condition is that over time t = 0 the market value is $n(0) = n_0$, which means we start to measure the time from the instant of purchase (when the purchase-sale transaction has been completed).

$$\frac{dn(t)}{n(t)} = -kdt \tag{3}$$

By integrating relationship (3) we arrive at $\left(n(t) \rangle 0\right)$

$\ln\left[n(t)\right] = -kt + \ln C,$ where $\ln C$ is the integration constant, then

$$n(t) = Ce^{-kt}$$

after substituting the initial condition $\left(t = 0, n = n_0\right)$ we obtain

$$C = n_0\text{, i.e.}$$

$$n(t) = n_0 e^{-kt} \tag{4}$$

Therefore the market value (price) of goods in relation to time behaves exponentially and from the value n_0 over time $t = 0$ falls asymptotically to zero. The depreciation of given goods derives from the fall in their price.[7] In practice this can be expressed as the difference between the initial and current status of the

[7] It possible to deal with the problem of quantifying the depreciation of products analogically by making a rough approximation with phenomena that can be carefully monitored and measured. One example of this might be the reduction in strength of materials when subjected to dynamic stress. Cyclic load changes on steel constructions cause metal fatigue, which is manifested as a reduction in the load-bearing capacity of the construction. This phenomenon can be described by the Wöhler curve, which resulted from research by the German railroad engineer Wöhler in the nineteenth century.

goods. Let us therefore define the value for absolute depreciation as $AO(t)$ at time *t*.

$$AO(t) = n_0 - n(t) = n_0 - n_0 e^{-kt} \tag{5}$$

It is often better to define the term for relative depreciation $RO(t)$ as a ratio of absolute depreciation $AO(t)$ for the initial price of the goods n_0.

$$RO(t) = \frac{AO(t)}{n_0} = \frac{n_0 - n(t)}{n_0} = 1 - e^{-kt} \tag{6}$$

A) We can now derive a differential equation for absolute depreciation. From the first part of relationship (5) it is clear that the absolute depreciation is $AO(t) = n_0 - n(t)$. The first derivative of absolute depreciation over time is $\frac{dAO(t)}{dt} = \frac{-dn(t)}{dt}$ and from relationship (2) it is then $\frac{dAO(t)}{dt} = kn(t)$. By expressing $n(t)$ using $AO(t)$ we derive $n(t) = n_0 - AO(t)$, which yields a differential equation for absolute depreciation, which appears as follows

$$\frac{dAO(t)}{dt} = k\left(n_0 - AO(t)\right)$$

This equation can be solved by separating the variables while respecting the initial conditions, i.e. $t = 0 \Rightarrow AO(0) = 0$, then $C = n_0$ and gives the solution expressed by the relationship (5).

B) Similarly we can also derive a differential equation for the relative depreciation $RO(t)$ we have already referred to, defined in relationship (6). It is obvious from relationship (6) that from $RO(t) = 1 - \frac{n(t)}{n_0}$ and the first derivative $RO(t)$ over time we obtain

$$\frac{dRO(t)}{dt} = \frac{d}{dt}\left[1 - \frac{n(t)}{n_0}\right] = -\frac{1}{n_0} * \frac{dn(t)}{dt} = \frac{1}{n_0} kn(t)$$

After introducing $n(t) = n_0 - AO(t)$ and $AO(t) = n_0 RO(t)$ we obtain the following differential equation for relative depreciation

$$\frac{dRO(t)}{dt} = k\left(1 - RO(t)\right)$$

By applying the method of separating the variables, we can again verify, while respecting the initial conditions. $t = 0 \Rightarrow RO(0) = 0 \Rightarrow C = 1$, that the equation for relative depreciation has the solution given in relationship (6).

2. Differential Equation of Price and Depreciation Dependent on Quadratic Time

In practice we often come across a case where the speed of the fall in the price of goods is proportionate not only to the price amount, but also to time. This means that we proceed on the basis of the equation

$$\frac{dn(t)}{dt} = -ktn(t) \tag{7}$$

This first order differential equation can again be solved by separating the variables. The initial condition is the same as in 1A), i.e. when $t = 0 \Rightarrow n(0) = n_0$. The individual stages of the solution follow the order below:

$$\frac{dn(t)}{n(t)} = -ktdt$$

from where by integration we find $\ln\left[n(t)\right] = \frac{-kt^2}{2} + \ln C$ and therefore

$$n(t) = Ce^{\frac{-kt^2}{2}}$$

Respecting the initial conditions, i.e. $t = 0 \Rightarrow n(0) = n_0$ then $C = n_0$ and the price is determined by the relationship $n(t) = n_0 e^{-k\frac{t^2}{2}}$

This gives absolute depreciation as

$$AO(t) = n_0 - n(t) = n_0 - n_0 e^{-k\frac{t^2}{2}} = n_0\left(1 - e^{-k\frac{t^2}{2}}\right) \tag{8}$$

Similarly relative depreciation is then

$$RO(t) = \frac{AO(t)}{n_0} = 1 - e^{-k\frac{t^2}{2}} \tag{9}$$

Similarly to part 1, we can derive differential equations for absolute and relative depreciation.

A) *Absolute Depreciation*

From the initial relationship $AO(t) = n_0 - n(t)$ it follows that the differential equation for absolute depreciation

$$\frac{dAO(t)}{dt} = \frac{d}{dt}\left[\left(n_0 - n(t)\right)\right] = -\frac{dn(t)}{dt} = ktn(t)$$

where n_0 is a constant and therefore $\frac{dAO(t)}{dt} = kt\left[n_0 - AO(t)\right]$, to give $\frac{dAO(t)}{n_0 - AO(t)} = ktdt$, we integrate this relationship to obtain a result in the form $\int \frac{dAO(t)}{n_0 - AO(t)} = k\frac{t^2}{2} + C_1$, from which we find that $AO(t) = n_0 - Ce^{\frac{-kt^2}{2}}$. The initial condition $t = 0 \Rightarrow AO(0) = 0$ defines $C = n_0$ and the resultant relationship for absolute depreciation is identical to relationship (8)

$$AO(t) = n_0\left(1 - e^{-k\frac{t^2}{2}}\right)$$

B) *Relative Depreciation*

With the relative depreciation defined by relationship (9) we derive analogically, as in paragraph 2A), the equation $\frac{dRO(t)}{dt} = \frac{1}{n_0}ktn(t)$, from which it follows that

$$\frac{dRO(t)}{dt} = \frac{kt}{n_0}\left[n_0 - AO(t)\right] = kt\left[1 - RO(t)\right]$$

Therefore the differential equation for relative depreciation appears as $\frac{dRO(t)}{dt} = kt\left[1 - RO(t)\right]$. Its solution by separating the variables is again reconciled with relationship (9) when the initial conditions are met $t = 0 \Rightarrow RO(0) = 0$ and $C = 1$.

3. Differential Equation of Price and Depreciation Dependent on Cubic Time

When conducting practical research into the fall in prices of goods, it is often not possible to assume that Case 1 always applies. (i.e., that the speed of price reduction is only proportionate to the amount of the price), nor that Case 2 applies. (i.e., that the speed of price decline is proportionate to the price and time), as it often depends on a combination of both.

Neither can we ignore the possibility that the speed at which prices fall is proportionate to the amount of these prices and some general function of time. In mathematical terms we can write this as

$\frac{dn(t)}{dt}=-kf(t)n(t)$, i.e. after separating the variables $\frac{dn(t)}{n(t)}=-kf(t)dt$, from where after integration we obtain $\int\frac{dn(t)}{n(t)}=-k\int f(t)dt+\ln C$, then $\ln\frac{n(t)}{C}=-k\int f(t)dt$, from which $n(t)=Ce^{-k\int f(t)dt}$, and then absolute depreciation

$$AO(t)=n_0-n(t)=n_0-Ce^{-k\int f(t)dt} \tag{10}$$

and relative depreciation

$$RO(t)=\frac{AO(t)}{n_0}=1-\frac{C}{n_0}e^{-k\int f(t)dt} \tag{11}$$

As an example of the use of these general formulae, we can present a case where[8]

$$f(t)=a_3t^3+a_2t^2+a_1t+a_0$$

and $\int f(t)dt=\int\left(a_3t^3+a_2t^2+a_1t+a_0\right)dt=\frac{a_3t^4}{4}+\frac{a_2t^3}{3}+\frac{a_1t^2}{2}+a_0t$

For absolute depreciation we obtain the formula $AO(t)=n_0-Ce^{-k\left(\frac{a_3t^4}{4}+\frac{a_2t^3}{3}+\frac{a_1t^2}{2}+a_0t\right)}$

[8] Further research indicates that the change in price may also be proportionate to a higher power of time than is cubically dependent, which can be expressed by developing a function $f(t)$ into the Taylor series in the area of point $t=0$ or by expressing the function $f(t)$ using the Taylor polynom, again in the neighborhood of point $t=0$, i.e. $f(t)=\sum_{k=0}^{\infty}\frac{f^k(0)}{k!}t^k$, or $T_n(f,t=0)=\sum_{k=0}^{n}\frac{f^{(k)}(0)}{k!}t^k$; $f^k(0)$ denotes the the *k*-th derivate of funkcion *f* evaluated at *0*.

Respecting the initial conditions $t = 0 \Rightarrow AO(t) = 0$ is $C = n_0$, the following equation for absolute depreciation is then $AO(t) = n_0 \left[1 - e^{-k\left(\frac{a_3 t^4}{4} + \frac{a_2 t^3}{3} + \frac{a_1 t^2}{2} + a_0 t\right)} \right]$

Similarly, relative depreciation is given in the relationship

$$RO(t) = \frac{AO(t)}{n_0} = 1 - e^{-k\left(\frac{a_3 t^4}{4} + \frac{a_2 t^3}{3} + \frac{a_1 t^2}{2} + a_0 t\right)}$$

Conclusion

How to conclude our discussion of the mathematical model?[9]

It can evoke discussions. It looks simple but in spite of its simplicity it is used in many scientific fields.

The above models can be considered linear in that the change in the number of units per time unit is proportional to the number of units considered. This is expressed by the differential equations for the time development of the value (price). The aim of further research is to formulate linear differential equations for individual classes of commodities which can find applications e.g. *in insurance*, where, inter alia, the prices of used goods are determined under the influence of the phenomenon of time. This model was used in simplified form by Česká pojišťovna during the floods of 2002.

This paper stems from a deterministic differential equation for prices which models and describes the development of prices and thus minimizes the influence of stochastic forces on setting estimates of prices. Only a first order differential equation has been used in our model. However if we want to include some stochastic forces in the model so that they will become part of the deterministic model, it will be necessary to use a differential equation of higher order. This equation will take into account effects such as changes in the of rate of growth or declining of prices. It was possible to fulfill this task only partially within our model beginning with equation (11) and subsequent relations. Within the framework of the model submitted this is shown by equation (11) and subsequent relations.

There are good reasons for this essay in conclusion emphasising its meaning and purposes in order to indicate its utilisation, or, as the case may be, interpretation. The objective is to contribute in one of many possible modelling of economic relations or processes the application of a mathematical apparatus which

[9] I respect religion but I believe in mathematics said Albert Einstein once speaking with Pius XII and he added: In your case, it would be the opposite. The Pope replied: Religion and mathematics are for me the manifestation of the same divine meticulousness. Taken from <http://www.knihovna.net/KNIHA/0042_t04.htm>.

is used not only in the exact sciences, but can be successfully applied in the social sciences. In this case the choice "fell" on the modelling of the principle of falling prices over time.

At the same time the theoretical standpoints of actual modelling are respected, monitoring the appropriate concentration and simplification of the fundamental phenomena of the problem in question. The theoretical standpoints of the magnitudes of value and price are also shared. A substantial part of the article presents a corroborative and predictive mathematical expression of the model. Another anticipated contribution of this essay is that it will offer ammunition for further exploitation within the sphere in question, issue a challenge for further theoretical discussion, but also act as an appeal for consideration to be given to the utilisation of a higher mathematical apparatus in economic and market practice.

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